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Articoli/2

*Leibniz and Modern Physics**

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We systematically analyze the conceptual structure of modern physics, i.e., the theory of relativity and quantum mechanics and quantum field theory, from the perspective of Leibniz's three basic principles of identity, sufficient reason and continuity. We find that Leibniz's way of thinking can still provide insight into fundamental problems of physics, such as the nature of physical constants, the relation between physical invariances and covariant coordinate representations, the role of the Planck scale and quantum mechanical indeterminism and the measurement problem. Leibniz himself could not possess all those insights, simply because physics was not yet sufficiently developed then, but he had already forged some conceptual tools with which some fundamental problems of modern physics can be addressed.

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Introduction

In his wide ranging thinking, Leibniz systematically employed three principles,

- the Principle of Identity (Law of Contradiction),
- the Principle of Sufficient Reason,
- the Principle of Continuity.

The Principle of Identity is the basis of his logic and a foundation of his metaphysics. An ontological subject, a monad in his later works, remains identical to itself in its development, as it carries its past and its future in itself. The Principle of Sufficient Reason stipulates that nothing exists without a proper cause. Consequences are that what is true can be demonstrated, that what cannot be distinguished is identical, and that physics is guided by conservation laws, the equipollence of cause and effect. The Principle of Continuity makes nature deterministic, unfolding like the solution of a differential equation, which can be computed with the tools of Leibniz' *calculus*[1]. For Leibniz, the ultimate

* Acknowledgement: I am grateful to Richard Arthur and Vincenzo De Risi for critical comments and extensive discussions which provided me with much insight.

principle of matter was not extension – contra Descartes –, but some intrinsic principle of change and resistance. For him, the ultimate constituents of reality were the monads with their active and passive forces[2]. Importantly, Leibniz's force is not Newton's force, but the active force was rather what he called *vis viva* and what is now called kinetic energy. From the equipollence of cause and effect, he deduced the fundamental law of physics, that of the conservation of energy, and from the principle of sufficient reason, he deduced optimality principles. It is still debated, however, to what extent these optimality principles were not only conceived as leading to preestablished harmony in the best of all possible worlds, but also more concretely to the physical principle of minimal (or better, stationary) action[3].

In a recent monograph[4], I have systematically explored the scope of these principles in Leibniz' thinking and what this thinking has to say for modern science. In the present contribution, I shall apply these principles to fundamental problems of modern physics. It will turn out that a Leibniz type thinking can still provide conceptual insight into the problems that the physics of the 20th century encountered.

Leibniz's own insights into physics derived from these – ultimately metaphysical – principles were overshadowed by the success of Newton's classical system of mechanics, as worked out by Euler (who, incidentally, had a profound antipathy against anything originating from Leibniz) and others. It was further perceived that Leibniz had lost the debate with the Newtonian Clarke about the nature of space and time. Likewise, it was thought that the critical philosophy of Kant had overcome the systematic flaws of the concept of monads of Leibniz and Wolff, and replaced the Leibnizian optimistic belief into the intelligibility of a rational world by the epistemological insight that the thing in itself was elusive and that space, time, and causality were a priori, but synthetic categories of the thinking subject. This began to change in the late 19th century. The discovery of non-Euclidean geometries posed a problem for Kant's conception of space as an a priori construction of the human mind. Leading physicists like Hermann von Helmholtz or Max Planck turned away from the Kantian paradigms and took a

[1] But this does not mean that we as humans can compute the future of the world from its current state, as would be possible in principle, although not in practice, in Laplace's conception. In Leibniz's view, the intrinsic laws of the monads and their states which depend on all the other simultaneously existing monads, are only known to God. Also, there are practical limits of computability. See for instance the analysis in R. Arthur, *Leibniz as a precursor to Chaitin's Algorithmic Information Theory*, in C. Meyns (ed.), *Information and the History of Philosophy*, London 2021, pp. 153-176. Also, as pointed out by R. Arthur, for Leibniz, such a determinism does not imply logical or metaphysical necessity.
[2] These concepts involve an intricate interplay between metaphysics and physics, and the distinction does not seem to be fully compatible with our current understanding of physics. For a systematic analysis, see H. Stammel, *Der Kraftbegriff in Leibniz' Physik*, Mannheim 1982.
[3] Aee J. Jost, *Leibniz and the calculus of variations*, in V. De Risi (ed.), *Leibniz and the structure of sciences: modern perspectives on the history of logic, mathematics, epistemology*, «Boston Studies in the Philosophy and History of Science», 337, pp. 253-270.
[4] J. Jost, *Leibniz und die moderne Naturwissenschaft*, Berlin 2019.

fresh look at Leibniz. Gottlob Frege developed his logic with explicit reference to Leibniz, although its systematic relational character went beyond what Leibniz himself had constructed. Leibniz's philosophical and mathematical writings became available through the editions of Gerhardt, a fact that facilitated a return to Leibniz for those people who became critical of Kantianism. At the beginning of the 20th century, Couturat[5] also edited the writings of Leibniz about logic, most of which were not published during Leibniz's life, but were only scattered in his manuscripts. At about the same time, Cassirer[6] and Russell[7] published systematic expositions of Leibniz philosophy, the latter one being rather critical, though, and claiming systematic contradictions in that philosophy. With the appearance of Einstein's theories of special and general relativity[8], interest in Leibniz gained further momentum, and, in particular, Hans Reichenbach[9] considered Einstein's theory as a vindication of Leibniz's conceptions of space and time. While Reichenbach, like Cassirer, had received his academic training in the Neokantian school which at the turn of the century was quite influential in Germany, he moved away from Kant and came to the conclusion that Leibniz's conceptions of space and, in particular, of time as a causal order were superior to those of Kant. Reichenbach's interpretation of Leibniz led to a critical discussion with Dietrich Mahnke from the phenomenological school of Edmund Husserl. In an impressive scholarly work[10], Vincenzo De Risi puts this debate into the context of Leibniz's theories of (meta)physics and analyzed where each of them (sometimes productively) misinterpreted Leibniz. To what extent Einstein's theory vindicates Leibniz against Newton is however still debated in the philosophy of physics. Similarly, philosophers of science interested in the foundations of quantum mechanics have not yet reached an agreement on whether Pauli's exclusion principle, that two different (fermionic) particles cannot share the same state, follows from Leibniz's principle of the identity of indiscernibles, which he had derived from the principle of sufficient reason. Nevertheless, both the intellectual level and the intensity with which Leibniz's principles and conclusions are currently discussed do not reach those of the earlier discussions. At least, this is the general picture. Still, there do exist

[5] L. Couturat (ed.), *Opuscules et fragments inédit de Leibniz*, Paris 1903; repr. Hildesheim 1966.
[6] E. Cassirer, *Leibniz' System in seinen wissenschaftlichen Grundlagen*, Marburg 1902; repr. Hamburg 1998.
[7] B. Russell, *The philosophy of Leibniz*, Cambridge 1900, 1937[2].
[8] A. Einstein, *Zur Elektrodynamik bewegter Körper*, «Annalen der Physik» 17/4, 1905, pp. 891-921; A. Einstein, *Die formale Grundlage der allgemeinen Relativitätstheorie*, «Preuss. Akad. Wiss. Sitzungsberichte», 1914, pp. 1030-1085; A. Einstein, *Die Feldgleichungen der Gravitation*, «Preuss. Akad. Wiss. Sitzungsberichte», 48-49, 1915, pp. 844-847 [Translated into English in *The Collected Papers of Albert Einstein*, vol. 6 (eds. A. Kox et al.) Princeton 1996].
[9] H. Reichenbach, *Die Bewegungslehre bei Newton, Leibniz und Huyghens*, «Kant-Studien», 29, 1924, pp. 416-438; H. Reichenbach, *Philosophie der Raum-Zeit-Lehre*, Berlin 1928
[10] V. De Risi, *Leibniz on Relativity. The Debate between Hans Reichenbach and Dietrich Mahnke on Leibniz's Theory of Motion and Time*, in R. Krömer, Y. Chin-Drian (ed.), *New Essays on Leibniz Reception in Philosophy of Science 1800-2000*, Basel/Boston 2012, pp. 143-185

original contributions, like that of Lee Smolin[11], who tries to utilize Leibniz's principle of sufficient reason and a relational interpretation of his monadology for developing a realist interpretation of contemporary physics with the ultimate hope of resolving its big enigma, i.e., the relation between quantum mechanics and the theory of relativity and how to unify them in a single theory.

In this contribution, I shall attempt to show that Leibniz's way of thinking can still provide important insight into deep problems of contemporary physics, in both relativity and quantum mechanics. I do not intend, however, to claim in any way that Leibniz was a forerunner of those theories. While Leibniz emphasized the relational character of space and time, he did not connect them through a finite speed of signal propagation. More fundamentally, he did not really resolve the tension between the nature of the monads whose states are unfolding through their internal laws and the interrelations between them. For the constraints on the simultaneous existence of monads, he had developed the notion of compossibility, and preestablished harmony as a selection principle, but he considered the web of relations between the monads as purely ideal. Thus, he developed a very advanced logic of substances and their properties, but the preceding conceptions may have prevented him from developing a systematically relational logic in the sense of Frege. Also, his distinction between active and passive force (*vis viva* vs. *vis mortua*) may have prevented him from achieving a unified picture of mechanics[12]. His principle of continuity, that he employed for letting his calculus compute the deterministic unfolding of dynamical processes from their initial values, did not leave room for the fundamentally discrete nature of quantum mechanics[13].

1. Physical Constants

According to Leibniz's principle of sufficient reason, there should be no physical constants, as there is no reason for fixing a particular absolute scale and such a scale could not be determined from observations, because «si toutes les

[11] L. Smolin, *Einstein's unfinished revolution. The search for what lies beyond the quantum*, New York 2019

[12] D. Bertoloni Meli, *Equivalence and Priority. Newton versus Leibniz*, Oxford 1993, argued that fundamental concepts of Leibniz's physics were developed in response to Newton's *Principia*, but H. Hecht, *Dynamik, Physik, Experiment*, in F. Beiderbeck, W. Li, S. Waldhoff (eds.), *Gottfried Wilhelm Leibniz. Rezeption, Forschung, Ausblick*, Stuttgart 2020, pp. 665-762, traced the systematic origins of Leibniz's physics in his early writings.

[13] Although R. Arthur, R. Arthur, *Leibniz and quantum theory. Lecture at Leipzig,* Nov. 2016, (available on his website) argued that Leibniz considered state changes as leaps that are so small that they remain below the threshold of discernibility (*Leibniz's metaphysics of change: vague states & physical continuity*, forthcoming in the *Festschrift* volume for Massimo Mugnai, F. Ademollo, V. De Risi, F. Amellini (eds.), *Thinking and Calculating. Essays on Logic, its History and its Applications,* to appear. For his systematic account of Leibniz philosophy of nature, see R. Arthur, *Leibniz,* Cambridge, Malden, MA 2014.

choses du monde qui nous regardent, estoient diminuées en même proportion, il est manifeste, que pas un ne pourroit remarquer le changement»[14].

In the same spirit, the standard model of elementary particle physics, while empirically successful, is often criticized on the grounds that it contains too many unexplained constants, and physicists therefore believe that it cannot represent the ultimate truth.

But I want to argue here, in line with Leibniz's principle, that some physical constants are only apparent. Let us consider the speed of light which is believed to be one of the basic and irreducible physical constants. *C* relates spatial and temporal units, and its value is given as approximately 300,000 kilometers per second. But the units employed here, kilometer and second, are arbitrary and do not represent any independent physical entities. In order to understand better what is happening here let us look at the three spatial dimensions. In principle, we could measure height and width in different units, as gravity causes an anisotropy of physical space. Then we would also get a constant relating those different units. But as we assume that spatial rotations constitute symmetries of space, we naturally put this constant equal to 1, that is, employ compatible measurements in the different spatial directions. But the same happens with space and time, according to Einstein's theory of relativity[15]. Spatial and temporal directions are naturally related by the propagation of light. Abstractly, the group of spatial symmetries consisting of rotations and translations is extended to the Poincaré transformations of space-time. That is, in Einstein›s theory of special relativity, physical laws are invariant under Poincaré transformations, and in particular under the Lorentz group of space-time rotations. That is, it is not that light propagates at a particular speed, but that light naturally relates spatial and temporal directions, and this can be used for defining speed. That is, putting the speed of light equal to 1 is not simply a convenient convention, but rather expresses a fundamental physical symmetry.

A conceptually similar argument applies to Newton's gravitational constant *g*, as it appears in the Einstein-Hilbert field equations[16]

$$R_{ij} - 1/2\, g_{ij}\, R = k T_{ij}\,, \qquad (1)$$

where R_{ij} is the Ricci tensor, and R is its trace, of the space-time metric g_{ij}. T_{ij} is the energy-momentum tensor that describes the distribution of masses in space-time, and it is coupled to the space-time geometry via

[14] *Leibnizens mathematische Schriften*, hrsg. v. C. I. Gerhardt, Bd. I, Berlin 1849; repr. Hildesheim/New York 1971, p. 180, in a letter to Galloys, that, according to a note of Leibniz, he did not send.

[15] A. Einstein, *Zur Elektrodynamik bewegter Körper*, loc. cit.

[16] They were derived by A. Einstein, *Die Feldgleichungen der Gravitation*, cit., and D. Hilbert, *Die Grundlagen der Physik (Erste Mitteilung)*, «Königl. Ges. Wissensch. Göttingen. Mathematisch-physikalische Klasse. Nachrichten», 1915, pp. 395-407. The priority is still being disputed, but that does not concern us here, and, in fact, these two scientists themselves did not pay much attention to this issue and remained on friendly terms.

$$k = 8\pi g/c^4, \tag{2}$$

where g is the gravitational constant and c is the speed of light. Again, g, like c, arises only from our conventions. The geometry of space-time and the forces in it are fundamentally linked; in fact, they are only two different ways of describing the same physics. The geometric curvature equals the physical force, and g and c simply translate the corresponding units of measurement.

2. Space and Time

As is commonly known, Leibniz debated about the nature of space and time with Clarke. Clarke defended Newton's concept of absolute space and time, whereas Leibniz's philosophy postulated that space and time as the order of things are relative, although not linked as in Einstein's theory of relativity. Nevertheless, Einstein's theory is frequently considered as a vindication of Leibniz's view, although the details of the debate are more subtle[17]. Since this is not a main issue of this paper, a brief summary might suffice. It seems that Leibniz did not yet realize all the systematic consequences embedded in his basic principles, and that there are some tensions in his work that are not easily resolved. While I think that the criticism of Russell went too far, there seems to be the problem between the windowless monads determined by their intrinsic laws or their internal logic, known only to God, and the relations between them. The pre-established harmony is an ingenious solution, but perhaps not fully satisfactory. In particular, it seems that he had difficulties reconciling his concept of space, quite modern in certain regards, with a perhaps still somewhat Aristotelian view of motion[18]. Einstein resolved this tension by saying that from the internal perspective of the moving object, it does not change its position, but rather everything else changes. Leibniz was perhaps quite close to that insight, but some remnants of old thinking may have prevented him from fully realizing it. Of course, this required to link space and time, and this link then makes the speed of light only an apparent additional constant. A photon has no time, and there cannot be separate natural units of space and time. Thus, like space, time only emerges from relations between beings, or in Leibniz's perspective, monads.

Here, however, I only want to discuss the following issues which in my view are in accord with Leibniz's thinking.

[17] see J. Jost, *Leibniz*, loc.cit., and the references given there.

[18] For a critical analysis, see V. De Risi, *Geometry and Monadology: Leibniz's Analysis Situs and Philosophy of Space*, Basel 2007, and V. De Risi, *Leibniz on the Continuity of Space*, in V. De Risi (ed.), *Leibniz and the structure of sciences: modern perspectives on the history of logic, mathematics, epistemology*, «Boston Studies in the Philosophy and History of Science» 337, pp. 111-169. For a defense of Leibniz, see R. Arthur, *Leibniz on Time, Space, and Relativity.* Oxford, forthcoming 2022.

Einstein's theory says that physical laws do not depend on a specific choice of coordinates. In other words, we can freely choose our frames of reference, that is, the perspective from which we describe physical events. This may remind us of Leibniz's view of the monads that all have their individual perspectives from which they represent the world. When in Einstein's theory the coordinates are changed, the equations (1) do not stay invariant, but transform according to the laws of Riemannian geometry[19]. More precisely, the quantities involved are tensors and obey the rules of Ricci's tensor calculus[20]. One speaks of covariance here. In contrast, in Newton's view, there is a distinguished frame of reference, absolute space and time. As was pointed out by E. Cartan[21] and K. O. Friedrichs[22], however, Newton's theory can also be formulated in a covariant manner, and when one formally lets $c \to \infty$, that is, decouples space and time, Einstein's equations reduce to Newton's law of gravitation. But then, the distinguished nature of absolute space and time in Newton's theory gets lost. In the covariant formulation, there no longer is a distinguished frame of reference. This has led to an interesting debate whether and if so, in which sense, Newton's original theory and its covariant version are equivalent. While Glymour[23] argues that the theories are not theoretically equivalent because Newton's original theory makes an ontological commitment that the covariant version does not make, Weatherall[24] in contrast modifies Glymour's criterion of theoretical equivalence by only requiring the equivalence in the sense of category theory[25], but not the isomorphism of the categories of models of two theories, with preservation of empirical content. Equivalence of categories is a weaker notion than isomorphism, and the difference is precisely that for the former notion, we may apply coordinate transformations, but not for the latter. Thus, I would argue that from Newton's perspective, one should go with Glymour's stricter criterion, according to which the theories are not equivalent, but from Leibniz' perspective, one should follow Weatherall and consider them as equivalent.

[19] B. Riemann, *Ueber die Hypothesen, welche der Geometrie zu Grunde liegen*, «Abh. Ges. Math. Kl. Gött.», 13, 1868, pp. 133-152; *Ueber die Hypothesen, welche der Geometrie zu Grunde liegen*, with commentary by J. Jost, *Klassische Texte der Wissenschaft*, Berlin/Heidelberg 2013; English version: *On the hypotheses which lie at the bases of geometry*, Basel 2016.
[20] G. Ricci, T. Levi-Civita, *Méthodes de calcul différentiel absolu et leurs applications*, «Mathematische Annalen», 54/1-2, 1900, pp. 125-201; see the modern presentation in J. Jost, *Riemannian geometry and geometric analysis*, Cham 2017.
[21] E. Cartan, *Sur les varietés à connexion affine, et la théorie de la relativité généraliseé (première partie)*, «Annales scientifiques de l'École Normale Superieure», 40, 1923, pp. 325-412; (suite), dto., 41, 1924, pp. 1-25.
[22] K. O. Friedrichs, *Eine invariante Formulierung des Newtonschen Gravitationsgesetzes und des Grenzüberganges vom Einsteinschen zum Newtonschen Gesetz*, «Mathematische Annalen», 98, 1928, pp. 566-575.
[23] C. Glymour, *The epistemology of Geometry*, «Nous», 11/3, 1977, pp. 227-251.
[24] J. O. Weatherall, *Are Newtonian Gravitation and Geometrized Newtonian Gravitation Theoretically Equivalent?*, «Erkenn.» 81 2016, pp. 1073-1091.
[25] For these concepts, see for instance J. Jost, *Mathematical concepts*, Cham 2015.

In fact, this is an instance of a more general principle, that of gauge invariance. The basic idea was first introduced by Hermann Weyl[26], and in the general version of Yang-Mills, it is a cornerstone of the standard model of elementary particle physics[27]. For instance, Maxwell's theory of electromagnetism can be formulated in terms of a vector potential (in mathematical terms, a connection) or in terms of the Faraday tensor (mathematically, the curvature of that connection). Different choices of the vector potential can give rise to the same Faraday tensor. This is the gauge freedom. The choice of the gauge does not affect the physical content, but only its formulation. In that sense, Newton's original formulation of his theory is not gauge invariant, but the covariant version is. Now in electromagnetism, there is no analogue of absolute space, that is, there is no particular gauge for which one could argue a distinguished status. Experimentally, this is confirmed by the Michelson-Morley experiment that eliminated the putative ether and that gave rise to Einstein's theory of special relativity. That theory does not yet involve gravity, but only electromagnetism, and with it, photons and the propagation of light. Here, as mentioned, we have covariance under Poincaré transformations. It was a difficult step for Einstein to include gravity and develop the general theory of relativity with convariance under arbitrary coordinate transformations.

3. The Hole Argument

There is a special case of the preceding discussion where people regularly invoke Leibniz's principles. This is the so-called hole argument. It was first conceived by Einstein[28], but subsequently not followed up by him, probably for good reasons. It was brought up again by Earman and Norton[29] and then intensively discussed by many philosophers of physics, see for instance[30]. The version of Earman and Norton is rather general and applies to any space-time theory with some differentiable manifold M and a couple of tensors $T_1, \dots, T_n$ that satisfy certain equations. These equations must be covariant in the sense that they transform appropriately under coordinate changes. Of course, the Einstein equations (1) satisfy these conditions. Local diffeomorphisms $h: M \to M$ can

[26] H. Weyl, *Raum, Zeit, Materie*, Berlin/Heidelberg 1918, 71988 (J. Ehlers, ed.).
[27] see for instance J. Jost, *Geometry and Physics*, Berlin/Heidelberg 2009.
[28] A. Einstein, *Die formale Grundlage der allgemeinen Relativitätstheorie*, «Preuss. Akad. Wiss. Sitzungsberichte» (1914), pp. 1030-1085.
[29] J. Earman, J. Norton, *What Price Spacetime Substantivalism? The Hole Story*, «Brit. J. Phil. Sc.» 38, 1987, pp. 515-525.
[30] A. Bartels, *Der ontologische Status der Raumzeit in der allgemeinen Relativitätstheorie*, in M. Esfeld (ed.), *Philosophie der Physik*, Frankfurt/M. 22012, pp. 32-49; M. Carrier, *Raum-Zeit*, Berlin/New York 2009; M. Carrier, *Die Struktur der Raumzeit in der klassischen Physik und der allgemeinen Relativitätstheorie*, in M. Esfeld (ed.), *Philosophie der Physik*, Frankfurt/M. 22012, pp. 13-31; T. Maudlin, *Philosophy of physics. Space and Time*, Princeton 2012; G. Macchia, *On spacetime coincidences*, in P. Graziani, L. Guzzardi, M. Sangoi (eds.), *Open problems in philosophy of sciences*, London 2012, pp. 187-216; J. O. Weatherall, *Regarding the 'Hole Argument'*, «British Journal for the Philosophy of Science», 69 (2018), pp. 329-350.

be seen as particular coordinate transformations, and since by assumption the field equations for the tensors transform appropriately, we cannot perceive any difference between the original theory and that transformed by *h*. In particular, *h* can be the identity outside some bounded subset *H* of *M*, called a hole. If we consider any coordinatization a model of the theory in question, then there are infinitely many models that agree outside *H*, but differ inside *H*. And no possible observation can distinguish between these models, as they are all equivalent, and only their coordinate description is different. And Earman and Norton then argue that in such a situation, one should better not be a Newtonian substantivist that grants reality to some or all those different descriptions, and rather accept Leibniz's arguments against Clarke in that regard. This seems undoubtably right. The point I want to make here is, however, that this is already incorporated in the concept of a differentiable manifold[31]. A differentiable manifold is defined as a maximal collection of compatible local coordinate descriptions. There is nothing substantivist contained in this concept. People that do not see this do not understand the concept of a differentiable manifold. In that sense, Leibniz' conceptual thinking is deeply embodied in that concept. As a consequence, Earman and Norton are right, but only make a point that has been understood and accepted in mathematics already for a long time, essentially since Hermann Weyl introduced the concept of a manifold[32].

In other texts, the hole argument is often presented somewhat differently. Maudlin[33] now argues that the diffeomorphism takes place on the very same background manifold and that the diffeomorphism then locates the same events differently on that background manifold. He argues that this is different from Leibniz's shift arguments and that this presents a serious problem. Unfortunately, the preceding should suggest that he does not seem to understand the concept of a differentiable manifold. There is no such thing as the very same background manifold with locations that remain invariant under the local diffeomorphism.

I have already argued[34], with a detailed presentation of the mathematics needed to understand this point, that in general relativity, we do not even have some time independent fixed topological background, as the so-called manifold substantivism suggests. Rather, the manifold evolves in time according to the Einstein equations from initial values on a so-called Cauchy hypersurface, and cosmologically therefore ultimately from initial conditions set by the big bang, at least as long as we are allowed to ignore quantum effects. A much more serious problem than the hole problem, and one which is not yet resolved, is whether solutions of the Einstein equations with initial values on a Cauchy hypersurface can lead to a so-called naked singularity, that is, one which in contrast to a black hole does not have an event horizon and which would therefore make the solution of the equations indeterminate. Penrose's cosmic censorship hypothesis,

[31] J. Jost, *Riemannian geometry and geometric analysis*, Cham 2017[7].
[32] H. Weyl, *Die Idee der Riemannschen Fläche*, Leipzig-Berlin 1913.
[33] T. Maudlin, *Philosophy of physics. Space and Time*, Princeton 2012.
[34] J. Jost, *Leibniz*, cit.

however, conjectures that no such solution exists[35]. As mentioned, this is still unsolved.

4. Quantum Physics and the Planck Scale

One of the early fundamental insights of quantum physics was the explanation of the photoelectric effect by Einstein[36]. A metal surface that is illuminated by light (of sufficiently short wave-length) emits electrons whose number depends on the intensity of the light source, but whose energy is independent of that intensity. Here, the light does not appear as a wave, but rather is composed of discrete particles, the photons, whose energy is quantized as

$$E = h\nu = h\,c/\lambda\ , \qquad (3)$$

where ν is the frequency and λ is the wave length of the light. h is Planck's constant, originally introduced by Planck[37] to explain black body radiation. Physicists prefer to work with $\hbar = h/2\pi$, as this accords better with certain conventions.

Again, it is not a meaningful question to ask why h has a particular value. Such a value depends on our units of measurement, that is, our conventions, and does not have any intrinsic physical meaning. There simply is the fundamental fact that at the quantum scale, nature no longer is continuous, but discrete, and everything comes in multiples of some unit. Analogously, when we count, we record items in terms of multiples of the basic counting unit 1, and it would be meaningless to ask why 1 has a particular value.

So far, everything is compatible with Leibniz's principle of sufficient reason. According to that principle, in a continuous world, there can be no natural scale, because no reason can be given for any particular value. This reasoning was important for Leibniz's approach to geometry[38]. Leibniz's criterion for the comparison of geometric figures was not congruence, but rather similarity, that is congruence up to scale. He argued that when one examined two figures, one could decide by intrinsic analysis of the individual figures whether they were similar, but not that they were congruent, because for the latter, one would have to superimpose one upon the other to see whether they match. As a

[35] R. Penrose, *The Question of Cosmic Censorship*, in R. Wald (ed.), *Black Holes and Relativistic Stars*, Chicago 1994, chap. 5; reprinted in «Journal of Astrophysics and Astronomy», 20, 1999, pp. 233-248.

[36] A. Einstein, *Über einem die Erzeugung und Verwandlung des Lichtes betreffenden heuristischen Gesichtspunkt*, «Annalen der Physik» 17/4, 1905, pp. 132-148.

[37] M. Planck, *Zur Theorie des Gesetzes der Energieverteilung im Normalspektrum*, Vortrag 14.12.1900, «Verhandl. Deutsche Physikalische Gesellschaft» 17, pp. 237-245.

[38] V. De Risi, *Geometry and Monadology: Leibniz's Analysis Situs and Philosophy of Space*, Basel 2007; V. De Risi, *Leibniz on the Parallel Postulate and the Foundations of Geometry. The Unpublished Manuscripts*, Basel 2016.

historical aside, Leibniz and his contemporaries were very much impressed by the investigations of Leeuwenhoek, Swammerdam, Hooke, and others with the newly invented microscope. Their observations seemed to demonstrate that there is life that is qualitatively similar to that of animals and plants at the much smaller microscopic scale. By extrapolation, it was then speculated that such forms of life also existed at still smaller scales, and their discovery only had to await sufficiently strong microscopes or other devices. Thus, the conclusion was that life had no intrinsic scale of magnitude. And Leibniz could have argued that when we enlarge or shrink everything by the same factor, then no difference could be perceived, as the perceiver would undergo the same change of scale. Only when the perceiver kept his scale and was then exposed to life at a different scale, a strange experience would happen, as in Gulliver's travels.

This argument, of course, no longer applies in a discrete world, where we do not measure, but rather count. While we cannot assign any meaningful scale to the basic unit, multiples of such a unit do have an absolute significance, and not only a relative one as continuous measurements that depend on an arbitrary choice of scale.

Now, remarkably, the existence of such a unit, together with the intrinsic symmetries of space, time and gravity that we have explained above, fixes scales for the basic physical quantities, length, time, and mass. I want to briefly explain that now, as it fits so well with Leibniz's principle of sufficient reason.

These are called the Planck length l_p, time t_p and mass m_p. We have identified three basic quantities, the speed of light c that relates spatial and temporal distances, the gravitational constant g that relates space-time to gravity, and Planck's constant h that sets the quantum scale.

According to (3), Planck's constant h has the physical dimension of an action, that is, energy times time. Since (kinetic) energy has the dimension mass times velocity squared, and velocity is simply length divided by time, when we write M for mass, L for length, and T for time, the dimension of is

$$[h]=(ML^2)/T \,. \tag{4}$$

That of as a velocity is

$$[c]=L/T \,. \tag{5}$$

Finally, the laws of Kepler and Newton imply that the force F in a gravitational field, that is, mass times acceleration, is given by $(gm_1\, m_2)/\mathrm{r}^2$ where m_1, m_2 are the masses of the two bodies involved and r is their distance. This implies

$$[g]=[F]\;\; L^2/M^2 = (ML/T^2)\;(L^2/M^2) = L^3/(MT^2) \tag{6}$$

We can therefore determine l_p, t_p, m_p by setting

$$\hbar=(m_P\, l_P^2)/t_P\,,\ \ c=l_P/t_P\,,\ \ g=(l_P^3)/(m_P\, t_P^2\,)\,. \tag{7}$$

We can solve these three equations for the three quantities l_p, t_p, m_p. We can express this in our conventional units of centimeters, seconds, and grams by inserting the experimentally determined values of c, g, $\hbar$ to obtain

$$l_P \approx 1.616\times10^{-33}\ \mathrm{cm} \tag{8}$$

$$t_p \approx 5.391\times10^{-44}\ \mathrm{s} \tag{9}$$

$$m_p \approx 2.176\times10^{-5}\ \mathrm{g} \tag{10}$$

The Planck length is very small, and the Planck time is very short, the time it takes light to travel the Planck length, but the Planck mass is quite large, about 10^{19} times the mass of the proton.

So far, everything is good, and compatible with Leibniz's law of sufficient reason. But now the problems emerge. Why is the mass of the proton so much smaller than the Planck mass? And why is the electron mass a particular small fraction of the proton mass? And why does Sommerfeld's fine structure constant α, the relativistic correction to atomic spectral lines (an electron in the first shell of the Rutherford-Bohr atomic model moves with $\alpha \approx 1/137$ times c), have a particular value? And more generally, why are there so many unexplained constants in the standard model of elementary particle physics? We currently do not see any sufficient reason for those, and (not only) from Leibniz's perspective, this is a basic problem of current elementary particle physics.

5. Quantum Mechanical Indeterminism and the Measurement Problem

The dynamics in a discrete world, like that of quantum physics, are fundamentally non-deterministic, because it consists of jumps between discrete states. In particular, Leibniz's law of continuity does not apply. I think that Leibniz saw this clearly, and he therefore did not believe in the possibility of such a discrete and indeterministic world. But one might ask what he would have said when confronted with such a physical world. Of course, this is a hypothetical question, but we can ask what his principles say in such a situation.

For that purpose, we should first briefly recall the principles of quantum mechanics. Quantum mechanical states are elements of some Hilbert space, like the space of square integrable complex valued functions, and the observables are operators on that Hilbert space. The eigenvalues of such an operator represent the possible outcomes of a measurement of the corresponding observable. The eigenvalue is the result of an operation on an eigenstate of that operator. Since the operators representing different observables in general do not commute, a state cannot simultaneously be an eigenstate for all of them, and therefore, not all observables can be measured simultaneously with arbitrary precision. This is quantified by the Heisenberg uncertainty principle.

There are two different schemes, the Heisenberg picture where the observables evolve and the Schrödinger picture where the states evolve. While these two schemes are mathematically equivalent, they may suggest different philosophical interpretations, or more precisely, different conceptions of the underlying reality. We shall present here the Schrödinger picture[39]. The evolution of the state of the system in question, perhaps the entire universe, is described by a wave function $\psi\ (x,t)$ that assigns a complex number to every point in space and every time . (The setting here is nonrelativistic, that is, space and time are not connected by the rules of special relativity. Relativistic quantum theory was introduced by Dirac and ultimately developed into modern quantum field theory[40], but the mathematical formalism is too heavy to be presented here.) Importantly, the wave function cannot be directly observed. But it evolves in time according to a fixed rule, given by Schrödinger's equation

$$\sqrt{(-1)}\ \hbar\ \partial\psi(x,t)/\partial t = H\psi(x,t)\ , \tag{11}$$

where H is the Hamiltonian of the system. This Hamiltonian is determined by a fixed rule, the correspondence principle, from its classical counterpart, the sum of kinetic and potential energy of the system. In the quantum mechanical picture, the Hamiltonian H is a second order differential operator, with derivatives taken w.r.t. the spatial variable x. As mentioned, the Schrödinger equation tells us how the wave function $\psi\ (x,t)$, which represents the state of the system, depends on the position x in space and evolves in time t.

The Schrödinger equation thus is a partial differential equation. In the simplest case when we have a particle (or, more precisely, its quantum mechanical analogue) of mass m that moves in a stationary potential $V(x)$, the equation becomes

$$\sqrt{(-1)}\ \hbar\ (\partial\psi(x,t))/\partial t = -\ \hbar^2/2m\ \ (\partial^2\ \psi(x,t))/(\partial x^2\ ) + V(x)\psi(x,t)\ . \tag{12}$$

Thus, the evolution of the wave function at different points is connected by spatial diffusion. The value of the state at every point instantaneously (because the setting here is nonrelativistic) influences the values at all other points. This is well in line with Leibniz's conception of a universal interdependence of all coexisting entities.

[39] For a mathematical presentation, one can take any textbook on quantum mechanics. For the wider context, see for instance J. Jost, *Geometry and Physics*, Heidelberg 2009. A presentation for more philosophically inclined readers can be found in T. Maudlin, *Philosophy of Physics. Quantum theory*, Princeton 2019.

[40] See for instance J. Jost, *Geometry and physics*, cit.

So far, everything looks unproblematic, simply describing a fundamental law of physics that is derived from some action principle (here, I do not enter into that, however, as it is mathematically too involved, and refer to the cited literature instead) and where the energy of the system plays a basic role. Action and energy were fundamental concepts of Leibniz' physics. The question, however, is how that relates to the observations that we can make, that is, to physical measurements. This is answered by Born's rule that stipulates that $|\psi(x,t)|^2$, the square of the absolute value of the complex valued quantity $\psi(x,t)$, is the probability density for observing the particle at time t at the position x. This is the problematic step. According to the picture that we have developed, we cannot predict where to find a particle at time t_1 even if we had known its exact position at some earlier time $t_0<t_1$, but we only have probabilities. Now these probabilities match the outcomes of all known quantum mechanical experiments, most famously the Stern-Gerlach single and double slit experiments. Nevertheless, there is the fundamental question why only probabilities, but not precise values of observables like the position of a particle evolve deterministically. Are we missing something here, that is, is the quantum mechanical description incomplete? After all, when we measure something, we find a definite value, that is, the particle is spotted at a precise location. The probabilities only emerge from repeated measurements. These probabilities thus concern possible values, but the question is which of them are actually realized, or more precisely, observed in a particular measurement. Now, while the result of a measurement cannot be determined, a measurement nevertheless yields a definite value, and not only some superposition of possible ones. The question how that could come about is called the measurement problem. Von Neumann and Wigner talked about a collapse of the wave function to some definite value, and the question has puzzled physicists ever since. One of the most radical proposals for a solution was brought forward by Everett[41]. His idea was to grant reality to all worlds that are possible outcomes of the Schrödinger equation, that is, all worlds that have a nonvanishing probability. At every moment, the, or better, any existing world branches into infinitely many possible successor worlds which thence no longer interact with each other. Thus, probabilities disappear, and whatever is possible becomes real. We, with our experience, just live in one of these branches, and other copies of us live in different branches, unbeknownst to us. Leibniz would have objected here that there should be some selection principle that makes only one of those possible worlds real, the best among the possible ones. And although it is still controversial to what extent he explicitly formulated this, when we express it in physical rather than in theological terms, this selection principle should be some kind of minimal action principle[42]. Remarkably, not

[41] H. Everett III, *"Relative State" Formulation of Quantum Mechanics*, «Reviews Modern Physics» 29, 1957, pp. 454-462.

[42] To be precise, one should speak of a principle of stationary, instead of minimal, action. Leibniz, in fact, was apparently aware of the possibility that the action need not necessarily be minimal, but only stationary. This point was missed by Maupertuis who claimed the universal-

only had Leibniz identified action as the basic physical quantity, and Planck's fundamental constant has precisely the dimension of an action, but a minimal action principle also is at the heart of Feynman's theory of path integrals[43], which is currently the most powerful scheme for performing quantum mechanical computations. Again, however, this only yields probabilities and does not contain a specific selection rule.

There is another aspect of the measurement problem. A quantum mechanical measurement measures the value of some quantum mechanical observable and records this on some macroscopic measurement device. That is, some transition from the quantum to the macro-world takes place. While a direct magnification of a quantum mechanical state seems conceivable as in Schrödinger's famous thought experiment of his cat, and also some experimental phenomena like the quantum Hall effect show such a transition, in general going from the quantum to the macro-world can be broken down into many intermediate steps where formally some asymptotic limit for some scale is going to infinity is taken. And when one takes such limits, not all formal properties of the lower scale are necessarily preserved. This is relevant, because in the arguments of von Neumann and Wigner, a collapse of the wave function was invoked to explain the loss of unitarity, a basic principle of the quantum world that is no longer obeyed in measurements. Von Neumann's dilemma is that linearity should lead to a superposition of measurement results when the quantum system is in a state of superposition. But many types of limits invoked in contemporary physics when going from the quantum to the nano-, thence to the micro- and finally to the macro-world, would not preserve such a feature as unitarity. Therefore, it seems natural to explain the phenomena seen in measurements also by taking suitable limits[44]. An analysis with refined tools from quantum field theory was recently given by Eriksson and Lindgren[45].

In any case, measurement seems to me an intrinsically physical operation, and therefore should be explained by a physical theory. It does not need any subjective component – after all, several physicists looking at the same measurement usually agree on the result. Invoking consciousness, as some quantum philosophers have attempted, leads to contradictions. Just imagine that Schrödinger's cat has been video recorded. Will its status then only be revealed when a conscious observer looks at the video? That does not make sense. At least, physicists now have a much better understanding up to which level

ity of his principle of minimal actions. In fact, in the discussion, people talked about maximal action, but this is misleading, as the action is not bounded from above. An important quantum mechanical computation device, Kramers's rule, requires an expansion about all stationary values. I do not enter here into the still ongoing controversy to what extent Leibniz had or at least could have anticipated the principle of least action as formulated by Maupertuis and Euler.

43 See R.Feynman, R. Leighton, M. Sands, *The Feynman Lectures on Physics*, Reading 1975

44 K. Hepp, *Quantum Theory of Measurement and Macroscopic Observables*, «Helvetica Physica Acta», 45, 1972, pp. 237-248.

45 K.-E. Eriksson, K. Lindgren, *Statistics of the Bifurcation in Quantum Measurement*, «Entropy» 21, 2019, 834, doi:10.3390/e21090834.

entanglement (a concept to be explained in more detail in the next section) can be maintained and what the difficulty is. Also, a measurement changes not only the quantum system, but also the measurement apparatus, and the latter perhaps more profoundly than the former, as it has a vastly larger number of quantum degrees of freedom. This issue is addressed in the theory of decoherence[46] and also in that of Eriksson-Lindgren[47].

6. Compossibility and existence

According to quantum mechanics, what is fundamental are probabilities, that is, partial existences, instead of absolute ones. In any case, existences need to satisfy compatibility conditions, like in the EPR phenomenon[48] of entanglement. We wish to argue that entanglement is some version of compossibility, even though Leibniz, not knowing quantum mechanics, of course could not conceive of it in that manner. We first need to explain the phenomenon itself. As in most discussions, we shall describe Bohm's version of EPR. For that, we need the concept of electron spin. An electron, in addition to its orbital angular momentum, the quantum version of the classical angular momentum, also possesses an internal angular momentum, called its spin. That spin has no classical analogue. An electron has total spin $\hbar/2$, and the resulting magnetic momentum exhibits itself in the outcomes of standard quantum mechanical experiments. The spin state of the electron is represented by a spinor, a vector with two complex numbers as entries and total norm 1. Again, the spin is an operator that operates on such states. More precisely, we have such a spin operator for each coordinate direction. Each such operator just has two eigenvalues which correspond to the spin being up or down in the corresponding direction. But since the operators for different directions do not commute, the spin cannot be measured in the three different spatial directions, denoted by x,y,z, simultaneously. The uncertainty principle implies that it can be measured only in one direction at a time. When the spin is fixed in the z-direction, for instance as up, then it is undetermined in the two other directions, that is, it could be up or down in the x- and the y-direction independently with equal probability 1/2. Now, however, we can pair two electrons so that the total spin vanishes. Thus, one of them has z-spin up, and the other z-spin down, but we may not know which. If we then let them fly into different directions so that they can no longer interact, but if we prevent them from interactions with other particles, and if we measure the z-spin of one of them and see that it is down, then we know that the z-spin of the other

[46] E. Joos, H. D. Zeh, C. Kiefer, D. Giulini, J. Kupsch, I.-O. Stamatescu, *Decoherence and the Appearance of a Classical World in Quantum Theory*, Berlin/Heidelberg 2003.
[47] K.-E. Eriksson, K. Lindgren, *Statistics of the Bifurcation in Quantum Measurement*, cit., p. 834.
[48] A. Einstein, B. Podolsky, N. Rosen, *Can quantum-mechanical description of physical reality be considered complete?*, «Phys. Rev.» 47, 1935, pp. 777-780; an extensive commentary by C. Kiefer is provided in A. Einstein, B. Podolsky, N. Rosen, *Kann die quantenmechanische Beschreibung der Realität als vollständig betrachtet werden?*, Berlin/Heidelberg, 2015.

is up, without having to measure it. This comes about because the total wave function of the system consisting of the two electrons entangles their spins. That is, only the combinations (up, down) and (down, up) are possible, and a measurement on one of them also determines the state of the other, even though they cannot interact or communicate in any conceivable way, if the experiment is done right. But something even more remarkable happens. When the electrons are thus entangled, such an anticorrelation occurs not only for the spin in the *z*-direction, but also for the spins in the other directions. Thus, if we measure the *z*-spin of one of them, and the *x*-spin of the other, then we know the spins of both of them in both the *z*- and in the *x*-directions, even though the uncertainty principle has told us that it is impossible to measure those two spins on any of them simultaneously.

Einstein, Podolsky, and Rosen had formulated the analogous argument for the position and momentum of two particles, which again, according to Heisenberg's uncertainty principle, cannot be measured simultaneously on a single particle. As just described in Bohm's spin version of this thought experiment, when two particles are entangled (a concept introduced by Schrödinger to explain this phenomenon), one can measure the position of one and the momentum of the other and thus determine the values of both observables for both particles simultaneously. Einstein, Podolsky, and Rosen thought that they had identified a basic problem of quantum mechanics, and leading quantum physicists, in particular Bohr, struggled in vain to account for it. As mentioned, Schrödinger introduced the concept of entanglement to account for it. Bell later reached the important conclusion that quantum physics is not local, because the phenomenon just described violated an inequality that, as he had found, would have to be obeyed by any local theory. And still later, experiments could confirm that the phenomenon really occurs[49]. And currently, it is at the basis of the emerging construction of quantum computers. It has turned into the technical problem to maintain the entanglement of sufficiently many particles for a sufficiently long time.

Back to Leibniz. When the particles in the universe are entangled with each other, then the values of some physical observable on one of them constrain the corresponding values for others. While in the above example of the two entangled electrons, each of them could have had the *z*-spin up, it was not simultaneously possible that both of them had that same value. Whether one wants to call that a preestablished harmony or give it some other name, the basic quantum physical reality is not local, but entangles different particles. Of course, there is the even deeper issue whether it is legitimate at all to speak of an underlying reality that is independent of any observer. Many quantum physicists of the 20th century had denied that. But if there is any such reality at all, it cannot be material in the sense of classical mechanics, but its basic constituents

[49] See for instance the accounts in A. Becker, *What is real?*, New York, 2018, or T. Maudlin, *Philosophy of physics. Quantum theory*, Princeton, 2019.

must be of a different nature, and they have to mutually constrain each other since not all combinations of their respective properties are compossible, as Leibniz had already argued.

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